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ECONOMICALLY EXPEDIENT PARAMETERS FOR HIGH-VOLTAGE "DEEP LEAD-IN" (LINES AND SUBSTATIONS) IN CITIES*†

A. A. GLAZUNOV, YU. L. MRZEL and T. A. KRUGLOVA

(Received 10 June 1976)

SUMMARY — Considers criterial analysis for all-round optimisation of the parameters by a per unit expenditure model for a network with two rated voltages. The parameters include substation siting and medium voltage circuitry. The trend to increasingly greater city power demand is anticipated in the next 15–25 years. A solution is proposed in terms of large 110 kV substations for radial lead-ins.

CITY growth is characteristic of our time in all countries, including development of the municipal-domestic economy and all spheres of human activity in cities. Such evolution of cities, their economy and production in their area is conditioned by considerable increases in demand for electric energy. In cities in U.S.S.R. and other countries the power-consumption increment indicators are higher than in many branches of the national economy; this trend will still continue in the next 15–25 years [1, 2 *et al.*].

The evolution of power consumption in cities is characterised by growth of the electric loads of all objects of development. As a result, in some Soviet cities the load density has reached 10 MW/km² and in the long term it can reach 40 MW/km².

Growth of the loads and consumption in cities requires careful study and prognosis of the expedient ways in which to develop urban supply systems.

The increasing role of high-voltage lines and substations (H.T. deep lead-in) has to be regarded as one of the main and characteristic features of present and long-term development of urban power supplies. This is linked with the economic expediency of this solution of supply problems with 10–15 MW/km² load density or more in large cities, and with the need to protect the ecological environment of city-dwellers [3].

**Elektrichestvo*, No. 4, 1–5, 1977.

†The article is based on research by A. Preis and the authors in the Electric Systems Faculty at Moscow Power Institute.

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**PROCEDURE FOR CALCULATING THE ELECTRODYNAMIC
STABILITY OF SUBSTATION CURRENT CONDUCTORS AT
110 kV OR MORE WITH REGARD TO "GIVE"
OF THE SUPPORTS***

YE. P. KUDRYAVTSEV and A. P. DOLIN

Moscow Energetics Institute

(Received 27 July 1976)

SUMMARY – Considers earlier work on estimating the electrodynamic stability of line conductors in the presence of increasingly great short circuit current. Hitherto error has occurred in design projection because of disregarding the "give" of the supports. It is due to "give" of the insulators and to deformation of their fastenings and bolt joints and it affects distribution gear at substations. A graphical method of analysis is proposed for any line voltage. Example.

INCREASING short-circuit current levels require quite accurate estimates of the electrodynamic stability of the current conductors. The simplifications common in design projection often lead however to considerable error. Conductor design for static load equal to the maximum electrodynamic force is still imperfect. If considerable "give" of the supports is present, appreciable error arises from having regard to bus vibration in the presence of a short circuit, but disregarding vibration of the insulators [1–3]. This problem arises above all in distribution gear at 110 kV or more. The "give" is due to the "give" of the insulators, to deformation of the structures to which they are fastened, and to movement of their bolt joints. Analysis of conductor electrodynamic stability in such distribution gear is made difficult by the lack of methods reducible to simple graphs.

In this paper a procedure is proposed for analysing the forces acting on the insulators and the stresses in the busbar material in the presence of a short circuit with

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regard to elastic oscillation of the system consisting of the busbar and insulators. The graphical calculations are simplified, and we discuss the results of research into how the electrodynamic stability of busbars and insulators is influenced by individual components of electrodynamic loads, some conductor parameters, the phase of connection and by the attenuation time constant of the short-circuit current aperiodic component.

As in [4], the bus can be regarded as a continuous beam with uniformly lengthwise distributed mass upon which a time-varying load acts in the presence of a short circuit. With equal distance between the insulators the spans of parallel bars are under the same conditions. It is therefore sufficient to consider one span.

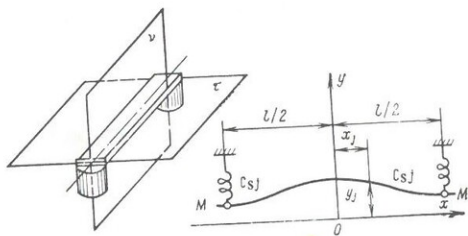


FIG. 1

The oscillations of the bus in its principal planes of inertia ν and τ (Fig. 1) are described by the differential equations [5]:

$$\frac{\partial^2 y_j}{\partial t^2} + \frac{EJ_j}{m} \frac{\partial^4 y_j}{\partial x^4} = \frac{q_j}{m}, \quad (j = 1, 2), \quad (1)$$

where t – time, sec; y_j – bus flexure, m; m – running mass of the bus, kg/m; EJ_j – bending stiffnesses, $H \cdot m^2$; q_j – electrodynamic loads per unit of bus length, H/m. The value $j = 1$ corresponds to bus oscillation in the plane of inertia ν , and $j = 2$ corresponds to oscillation in the τ plane. In any arrangement of mutually parallel phases and any bus-support orientation in relation to the planes ν and τ in the presence of a short circuit, loads arise which are uniformly distributed over the length of the bus and are equal to:

$$q_j = \frac{\alpha}{a} I_m^2 \sum_{n=1}^6 D_{jn} T_n(t), \quad (2)$$

where $\alpha = 2 \cdot 10^{-7}$, H/A^2 ; a – parameter characterizing the distances between buses, m; I_m – amplitude of the periodic component of the short circuit current, A;

$$\left. \begin{aligned} T_1 &= 1; T_2 = e^{-2t/T_a}; T_3 = e^{-t/T_a} \cos \omega t; \\ T_4 &= e^{-t/T_a} \sin \omega t; T_5 = \cos 2\omega t; T_6 = \sin 2\omega t. \end{aligned} \right\} \quad (3)$$

Here T_a – attenuation time constants of the aperiodic component of the short-circuit current, sec; ω – current angular frequency, rad/sec.

The values of the constant coefficients D_{jn} ($j = 1, 2; n = 1, 2, \dots, 6$) are determined by the mutual arrangement of the conductor buses, support orientation, type of short circuit and the short-circuit current phase of connection ψ . These have already been found for some types of conductor [1, 6].

Each of the functions (3) can be written in complex form:

$$T_n(t) = b_n e^{(\beta_n + i\gamma_n)t} + d_n e^{(\beta_n - i\gamma_n)t}, \quad (4)$$

where b_n, d_n, β_n and γ_n – constants expressed in terms of parameters of functions (3). Thus the conductor electrodynamic-stability estimation problem for quite complex loads (2) is reducible to addition of solutions of eqns (1) with right-hand sides of the kind (4) [7].

The full solution of differential equation (1) with right-hand side (4) will be the sum of its particular solution $\bar{y}(x, t)$ and the total solution of eqn (1) without the right-hand side $\bar{\bar{y}}(x, t)$:

$$y(x, t) = \bar{y}(x, t) + \bar{\bar{y}}(x, t). \quad (5)$$

We shall look for the particular solution of eqns (1) with right-hand side (4) in the form

$$\bar{y}(x, t) = \sum_{s=1}^2 \bar{Y}_s(x) e^{[\beta - (-1)^s i\gamma]t}. \quad (6)$$

We omit the subscripts indicating the number of the principal plane of inertia and the number of the load component (3) in formula (6) and throughout. By setting (6) in (1), we arrive at the ordinary linear differential equation with constant coefficients in the functions \bar{Y}_s :

$$\frac{d^4 \bar{Y}_s}{dx^4} + \frac{m}{EJ} [\beta - (-1)^s i\gamma] \bar{Y}_s = U_s, \quad (7)$$

where U_s – constants. Particular solutions of eqn (7) with the right-hand side are also constants. Arbitrary constant solutions of eqns (7) without the right-hand side can be found from the boundary conditions: for $x = \pm l/2$

$$\frac{\partial \bar{y}}{\partial x} = 0; \quad (8a)$$

$$2EJ \frac{\partial^2 \bar{y}}{\partial x^2} - c_s \bar{y} - M \frac{\partial^2 \bar{y}}{\partial t^2} = 0, \quad (8b)$$

where l – bus span length, m ; c_s – support stiffness, H/m; M – support referred mass, kg. Usually the axes of the insulators lie in one of the principal planes of inertia of the busbars. In Fig. 1 this plane is the plane v . In the presence of oscillation in this plane the referred mass M can be assumed equal to the mass of the support. If the oscillations occur in the plane τ , the referred mass can be found e.g. by the formula

$$M = \frac{c_s}{\Omega_s^2}, \quad (9)$$

where Ω_s – support angular frequency of natural oscillation, rad/sec.

Condition (8a) requires absence of bus-support section turns, and condition (8b) is a precondition for equilibrium of the unit for fastening the bus to the insulator. In this condition the addend $2EJ \partial^2 \bar{y} / \partial x^2$ is the sum of the transverse forces in the bus to the left and right of the support cross-section; the addend $c_s \bar{y}$ is the bus-insulator force of interaction proportional to the insulator-head displacement, the addend $M \partial^2 \bar{y} / \partial t^2$ has regard to the inertia of the mass of the support.

The total solution $\bar{y}(x, t)$ of a homogeneous equation which, like solution (6), satisfies boundary conditions (8), is obtained as a series in the eigen functions [5]:

$$\bar{y}(x, t) = \sum_{k=1, 3, 5, \dots}^{\infty} X_k(x) (L_k \sin \Omega_k t + N_k \cos \Omega_k t). \quad (10)$$

It can be shown that the eigen functions X_k are equal to

$$X_k = \frac{\operatorname{ch} \frac{r_k x}{l}}{\operatorname{sh} \frac{r_k}{2}} + \frac{\cos \frac{r_k x}{l}}{\sin \frac{r_k}{2}} \quad (11)$$

where r_k – parameters of the oscillation natural frequencies of the elastic "bus-insulator" system, given by the transcendental equation

$$\left(\frac{c_s l^3}{EJ r_k^3} \frac{M}{ml} r_k \right) \left(\operatorname{cth} \frac{r_k}{2} + \operatorname{ctg} \frac{r_k}{2} \right) - 4 = 0. \quad (12)$$

The roots of eqn (12) were calculated by digital computer. The curves obtained as a result for the parameter r_1 are given in Fig. 2.

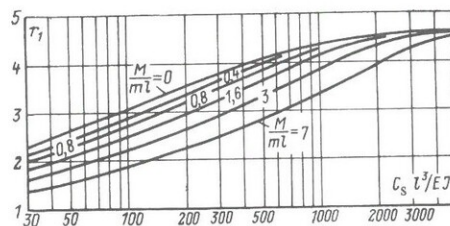


FIG. 2.

The angular frequencies of the natural oscillation Ω_k (rad/sec) are

$$\Omega_k = 2\pi f_k = \frac{r_k^2}{l^2} \sqrt{\frac{EJ}{m}}. \quad (13)$$

Here f_k – natural frequencies of bus oscillation on flexible supports, c/s.

The constants L_k and N_k are found from the initial conditions:

$$y|_{t=0} = 0; \quad \frac{\partial y}{\partial t} \Big|_{t=0} = 0. \quad (14)$$

In practical analysis it is sufficient to require the initial conditions (14) to be satisfied exactly only in some of the bus cross-sections. Thus, for instance, if the number of such sections is p , then conditions (14) give a set of $2p$ linear algebraic equations to find L_k and N_k .

The foregoing formulae enable bus and insulator flexures to be determined, whether due to individual components of the electrodynamic load, or to the load (2) as a whole. The mutual arrangement and orientation of the buses can be arbitrary. The analysis is suitable to a wide class of current conductors.

The loads on the insulators in Newtons and the maximum flexural stresses in the bus sections in Pascals are given by the formulae

$$R(t) = c_s y(l/2, t); \quad \sigma(x, t) = \frac{EJ}{W} \frac{\partial^2 y(x, t)}{\partial x^2}, \quad (15)$$

where W – bus cross-section moment of resistance in bending, m^3 . The solutions (15), when considered with (2), (5), (6) and (10), are reducible to the form

$$R = \frac{\alpha l}{a} I_m^2 R_*(t); \quad \sigma = \frac{\alpha l^2}{12aW} I_m^2 \sigma_*(x, t), \quad (16)$$

where R_* and σ_* – relative loads on the insulators and the relative stresses in the bus material.

We will now turn to the case where the bus is under the electrodynamic load (2). The loading on the insulator in the principal plane of inertia with number j is in this complex; we denote it as $R_{j\Sigma}(t)$. We put $\sigma_\Sigma(x, t)$ for the maximum stress in the bus material. The relative force acting on the insulator when only one of the components of the electrodynamic loads (2) with number n acts only in one principal plane of inertia j , is denoted by R_{jn} . For the relative stresses with this loading we use the notation

σ_{jn} .

Given R_{jn} and σ_{jn} , the overall loads $R_{j\Sigma}$ on the insulator and the stresses σ_Σ are given by the formulae

$$R_{j\Sigma}(t) = \frac{\alpha l}{a} I_m^2 R_{j\Sigma}^*(t); \quad \sigma_\Sigma(x, t) = \frac{\alpha l^2}{12aW} I_m^2 \sigma_{j\Sigma}^*(x, t), \quad (17)$$

where the relative loads $R_{j\Sigma}^*$ are

$$R_{j\Sigma}^* = \sum_{n=1}^6 D_{jn} R_{jn}^*(t), \quad (18)$$

whilst the relative stresses $\sigma_{j\Sigma}^*$ in the material of busbars of circular cross-section, as used in distribution gear at 110 kV or more, are defined as follows:

$$\sigma_{j\Sigma}^*(x, t) = \sqrt{\sum_{j=1}^2 \left[\sum_{n=1}^6 D_{jn} \sigma_{jn}^*(x, t) \right]^2}. \quad (19)$$

Especially important are the maximum absolute values of the functions $R_{j\Sigma}^*$ and $\sigma_{j\Sigma}^*$. Just as in [1], we will refer to them as dynamic coefficients:

$$\max |R_{j\Sigma}^*(t)| = \eta_{jR}; \quad \max |\sigma_{j\Sigma}^*(x, t)| = \eta_{j\sigma}.$$

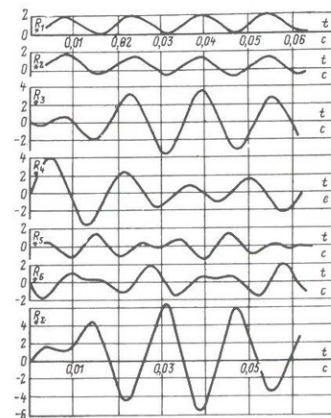


FIG. 3

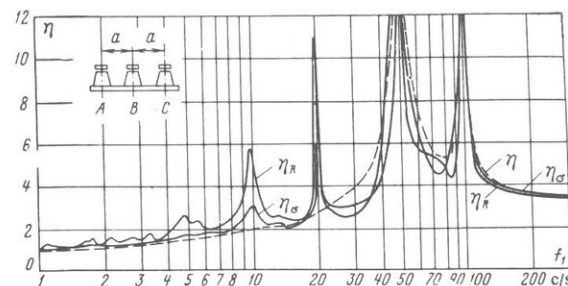


FIG. 4

The above method was applied by Minsk-32 digital computer to calculate the relative loads on insulators and the relative stresses in different cross-sections of parallel busbars laid in the same plane (see Fig. 4). The parameters of the conductors were varied in the following range in the computation:

$$0 \leq M/ml \leq 7, \quad 25 \leq c_s l^3/EJ \leq 50\,000, \quad 0.05 \leq$$

$$T_a \leq 0.2 \text{ sec}, \quad 1 \leq f_1 \leq 500 \text{ c/s}, \quad 0 \leq \psi \leq 2\pi \text{ rad.}$$

Two- and three-phase short circuits were considered. Six terms of the series (10) were retained.

By way of example, Fig. 3 shows the relative loads $R_n(t)$ on the insulator ($n = 1, 2, \dots, 6$) from the solution of eqn (1) for:

$$M/ml = 10; c_s l^3/EJ = 1500; T_a = 0,05 \text{ sec and } f_1 = 65 \text{ c/s.}$$

For the values of the parameters taken here, the plots in Fig. 3 can be regarded as ratings. They can be used with given constants D_n to find the loads on supports for conductors of different configuration. The lower curve in Fig. 3 indicates the insulator loads in the presence of a two-phase short circuit. In this case the non-zero load parameters were assumed in accordance with (1) to be:

$$D_1 = 0,5; D_2 = 10; D_3 = -20; D_5 = 0,5.$$

These load parameters (2) correspond to the angle $\psi = 0,5\pi$ at which the electrodynamic load becomes maximum [subscript j of the $R_n(t)$ functions and constants D_n is omitted because oscillation in the same principal plane of inertia is considered in the example]. The maximum insulator load is here reached in 0.031 sec from onset of the short circuit.

By varying the parameters M/ml and $c_s l^3/EJ$, and also ψ , T_a and f_1 , a study can be made of how they influence the maximum loads on the insulator and the stresses in the material of the busbars. Figure 4 treats the case where

$$M/ml = 0; c_s l^3/EJ = 10000; T_a = 0,05 \text{ sec and } \psi = 0,5\pi.$$

In this diagram the dynamic coefficients of the insulator load and busbar stresses in the presence of a two-phase short circuit are related to the fundamental natural frequency of oscillation of the bus-insulator system.

Calculations showed that over a wide range of parameter values analysis of a busbar as a beam with distributed mass is closely similar to bus analysis on the basis of a simpler model with one degree of freedom.

Consider now a flexible system with one degree of freedom. Its motion in one of the principal planes of inertia is described by the equation:

$$m_r \frac{d^2 y_r}{dt^2} + c_r y_r = F_r, \quad (20)$$

where m_r , y_r , c_r and F_r are the referred mass, flexure, stiffness and electrodynamic force respectively. The solutions of eqns (1) and (20) are closely similar if the referred parameters of the scheme with one degree of freedom are determined from the following conditions:

$$F_r = ql; \quad (21a)$$

$$\frac{1}{c_r} = \frac{1}{c_r} + \frac{1}{c_{r,b}}; \quad (21b)$$

$$f_r = f_1, \quad (21c)$$

where $c_{r,b} = 384 EJ/l^3$ – referred stiffness of the busbar, N/m; f_r – referred frequency of the system, c/s. Condition (21b) requires equality of bus-flexure at the mid-point of the span and displacement y_r of the scheme under static loads of the same magnitude. Condition (21c) requires equality of the fundamental natural frequency f_1 of the conductor oscillation to the natural frequency f_r of the scheme. These two conditions single-valuedly “fix on” the value of the referred mass because

$$m_r = c_r / (2\pi f_r)^2.$$

The general solution of eqn (20) is given, for example, in [5]. In use of the scheme with one degree of freedom the insulator load and the bus support-section stresses are given by the formulae:

$$\left. \begin{aligned} R(t) &= c_s y_s(t); \\ \sigma(t) &= \frac{M}{W} = \frac{c_{r,b} l}{12W} y_b(t), \end{aligned} \right\} \quad (22)$$

where M – bending moment.

The maximum loads R_{\max} and stresses σ_{\max} can be written so:

$$R_{\max} = \frac{\alpha l}{a} I_m^2 \eta; \quad \sigma_{\max} = \frac{\alpha l^2}{12aW} I_m^2 \eta, \quad (23)$$

where η – dynamic coefficient.

For a fully defined electrodynamic load the dynamic coefficient η is a function of only one natural frequency f_1 of the scheme. For the selected type of conductor and known short circuit, one and the same plot can be used for any support stiffness and insulator mass. This imparts considerable universality to calculations by formula (23). Formula (13) can be used to find the values of frequency f_1 necessary for the analysis.

One of the solutions of eqn (20) for the initial conditions (14) with the referred parameters selected in accordance with (21) was compared with the more precise solution of eqn (1) in Fig. 4. The results of the comparison can be regarded as satisfactory. Notable divergence only occurs at low natural-oscillation frequencies f_1 . Some improvement can be obtained in the agreement between the curves by applying a correction to condition (21a), i.e. by variation of the referred load.

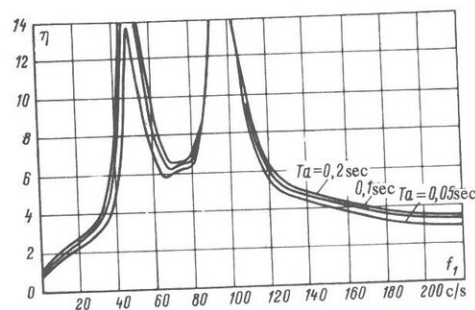


FIG. 5

Figure 5 plots the dynamic-coefficient curves $\eta = f(f_1)$ in the presence of a three-phase short circuit for parallel busbars laid in the same plane and time constants T_a equal to 0.05, 0.1 and 0.2 sec. The η -plots in this diagram are the envelopes of the dynamic coefficient curves for the middle bus and the extreme buses.

The research enables the following conclusions to be drawn.

With increasing "give" of the conductor supports and increasing mass of the insulators the natural oscillation frequencies of the bus-insulator system decrease and then the resonance regions are shifted and deformed. By varying the stiffness of the supports, one can thus influence the oscillation regime and "de-tune" the conductors from resonance.

The regions of conductor parameters in which greater vibration is possible in operation as well as during short circuits are best avoided owing to the reduction of electrodynamic stability in these regions and the greater vibro-noise activity of the conductors. This justifies use of eqns (1) and (20) which disregard energy dissipation in oscillation. Regard for dissipative forces has a notable effect on the results only in the resonance regions.

The resonance regions corresponding to higher forms of natural oscillation are narrow and so de-tuning from resonance is facilitated here. At natural frequencies of the bus-insulator system above 25 c/s the values of $R_{j\Sigma}$ and σ_{Σ} practically coincide over the whole of the investigated range of the $c_s l^3/EJ$ and M/ml parameters at the same natural frequencies. Appreciable differences only obtain at low f_1 frequencies.

At frequency greater than 200–300 c/s the maximum stresses in the busbars and the insulator loads are approximately equal to those stresses and loads which occur when the maximum electrodynamic forces are the static load of the busbars.

If the insulators possess relatively great stiffness ($c_s l^3/EJ > 15,000$ to 30,000), which is usually the case in distribution gear at voltages up to 35 kV, the analysis can be made without regard to the "give" of the supports by the method presented e.g. in [3].

If the busbars are arranged in the same plane, the maximum loads on the insulators and stresses in the bus material are greater on the middle phase when the conductor natural oscillation frequency is more than 50 c/s, but greater on the extreme phase when this frequency is below 50 c/s.

At conductor natural oscillation frequencies close to 50 c/s the dynamic coefficients increase rapidly with increase of the time constant T_a .

In all the calculations performed for the make phase in which the electrodynamic loads reach their greatest value, the insulator loads and busbar stresses reach values close to maximum. The deviations were no more than 10 per cent.

Example. It is required to find the maximum insulator loads and busbar stresses for an experimental conductor design with stiff 110 kV load [8].

Initial data. Busbars of annular cross-section are arranged in the same plane; bus outside diameter $D = 30$ mm, inside diameter $d = 20$ mm; bus material modulus of elasticity $E = 7 \cdot 10^4$ N/mm²; running mass of bus $m = 1.06$ kg/m; span length $l = 7.65$ m; distance between axes of adjacent phases $a = 1$ m.

Support stiffness $c_s = 1960$ N/mm (including insulator, bolts and base); support natural oscillation frequency $f_s = 58.9$ c/s. Acting value of the short-circuit current periodic component $I_p = 7.5$ kA; attenuation time constant T_a of the short-circuit current aperiodic component is 0.095 sec.

Solution. In accordance with expression (9), find the referred mass of the insulator:

$$M = \frac{c_s}{(2\pi f_s)^2} = \frac{1960 \cdot 10^3}{(2\pi \cdot 58.9)^2} = 14.33 \text{ kg.} \quad (\text{A/1})$$

The moment of inertia and moment of resistance of the cross-section of a bus of circular section are calculated by the formulae:

$$J = \frac{\pi}{64} (D^4 - d^4) = \frac{\pi}{64} (3^4 - 2^4) 10^{-8} = 3.19 \cdot 10^{-8} \text{ m}^4;$$

$$W = \frac{\pi}{32} (D^3 - d^3) = \frac{\pi}{32} (3^3 - 2^3) 10^6 = 1.86 \cdot 10^{-8} \text{ m}^3. \quad (\text{A/2})$$

And the parameters of the bus-insulator system:

$$\frac{M}{ml} = \frac{14,33}{1,06 \cdot 7,65} = 1,77; \quad (A/3)$$

$$\frac{c_s l^3}{EJ} = \frac{1960 \cdot 10^3 \cdot 7,65^3}{7 \cdot 10^{10} \cdot 3,19 \cdot 10^{-8}} = 392962,6.$$

Using the curves in (Fig. 2) for the M/ml and $c_s l^3/EJ$ values in question find the frequency parameter $r_1 = 4,73$. Then by formula (13) calculate the natural oscillation frequency of the bus-insulator system:

$$f_1 = \frac{4,73^2}{2\pi \cdot 7,65^2} \sqrt{\frac{7 \cdot 10^{10} \cdot 3,19 \cdot 10^{-8}}{1,06}} = 2,79 \text{ c/s.} \quad (A/4)$$

We now avail ourselves of the simplified procedure based on solving the oscillation problem of a system with one degree of freedom.

For the calculated natural oscillation frequency and attenuation time constant $T_a = 0,095$, the dynamic coefficient of a three-phase short circuit is found from the curves in Fig. 5 to be $\eta = 1$.

The maximum insulator load and busbar stress are calculated by formulae (23):

$$R_{\max} = \frac{2 \cdot 10^{-7} \cdot 7,65}{1} [\sqrt{2} \cdot 7,5 \cdot 10^3]^2 \cdot 1 = 172,1 \text{ N;} \quad (A/5)$$

$$\sigma_{\max} = \frac{2 \cdot 10^{-7} \cdot 7,65^2}{12 \cdot 1 \cdot 1,86 \cdot 10^{-6}} [\sqrt{2} \cdot 7,5 \cdot 10^3]^2 \cdot 1 = 59 \cdot 10^6 \text{ Pa} = 59 \text{ MPa.} \quad (A/6)$$

For a two-phase short circuit which has been experimentally investigated [8], the dynamic coefficient is $\eta = 1,2$, whilst the calculated maximum stress in the bus material is $\sigma_{\max} = 70,8 \text{ MPa}$ ($1 \text{ MPa} \approx 10 \text{ kgf/cm}^2$). The experimentally obtained maximum stress was $\sigma = 64,5 \text{ MPa}$. So the error of the calculations is 9.75%. At the same time the measuring error is $\pm 10\%$.

Thus the proposed procedure agrees quite well with experimental data (given e.g. in [8]), it is convenient for engineering calculations, it is applicable in project design in practice for calculating the electrodynamic stability of conductors of any voltage, and it is not inferior in accuracy to more cumbersome calculations developed abroad.

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